X-RAYS.

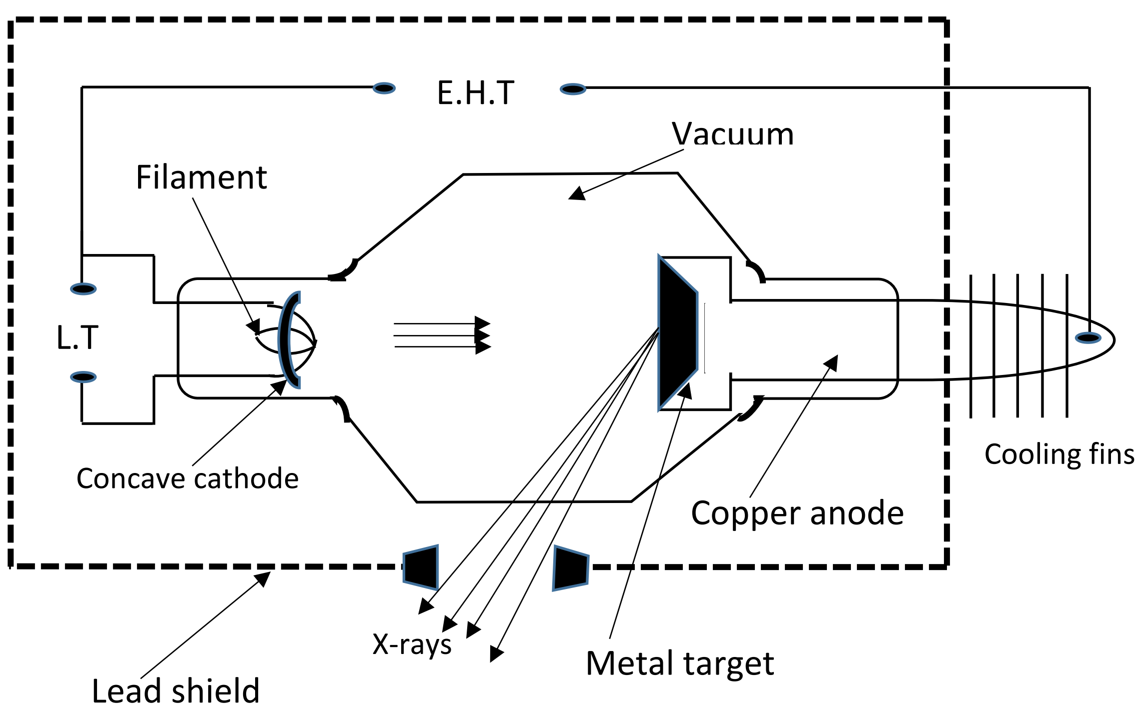
X-rays are electromagnetic radiations produced when fast moving electrons are stopped by matter.

Properties of X-rays.

(i) They are electromagnetic radiations.
(ii) They carry no charge.
(iii) They readily penetrate matter.
(iv) They affect the photographic plate.
(v) They can cause photo-electric emission to occur.
(vi) They are not deflected in both electric and magnetic fields.
(vii) They ionize gases through which they pass.

PRODUCTION OF X-RAYS.

X-rays can be produced in x-ray tubes, which are widely used in medical treatment. The figure below shows the essential features of an x-ray tube.



Mode of action.

Current produced by the low-voltage (L.T) source flows through the filament which makes the cathode hot. The hot cathode emits electrons thermionically. The emitted electrons are accelerated to high speeds through vacuum, by the high operating tube voltage (E.H.T), towards the anode. On striking the target, about 99% of the kinetic energy of the electrons goes into heat production at the target, conducted away by the copper anode and dissipated to the surroundings by the cooling fins, while the remaining percentage goes into x-ray production.

NOTE:

Since about 99% of the electron energy is converted into heat, then the target must be made of material with a high melting point, such as tungsten, usually embedded into the copper anode.

The heat generated at the anode is conducted away by the cooling fins or by a circulating liquid such as oil.

The presence of vacuum is to avoid collisions between the fast moving electrons and the air molecules.

The intensity of x-rays is controlled by changing the filament current, while the penetrating power (quality) of the x-rays is controlled by the operating tube voltage. X-rays produced with a low penetrating power are called soft x-rays, while those produced with a high penetrating power are called hard x-rays.

If there are n electrons which strike the target in one second, then the electron current, I will be given by;

$$\mathbf{I = ne}$$

Where **e** is the electronic charge, equal to $1.6\ x\ 10^{-19}\ C$.

Uses of X-rays.

(i) They are used in medicine to;
- locate bone fractures
- destroy cancer cells
- sterilize medical equipment
- detect lung tuberculosis.

(ii) They are used to locate internal imperfections in welded joints.
(iii) They are used to study the structure of crystals.
(iv) They can be used to detect firearms at international airports.

EXAMPLES: (Skip some spaces for the solutions)

1. Calculate the wavelength of the most energetic x-rays produced by a tube operating at 1.0 x $10^5 V$. $[Use\ Planck's\ constant = 6.6\ X\ 10^{-34} Js,$ $electronic\ charge = 1.6\ X\ 10^{-19} C,\ \ speed\ of\ light = 3.0\ X\ 10^{8} ms^{-1}]$
$(Ans \quad \lambda = 1.24\ x\ 10^{-11} m)$

2. The most energetic x-rays produced by a particular x-ray tube have a wavelength of 2.1 X $10^{-11} m$. What is the accelerating p.d of the tube, given that h = 6.6 X $10^{-34} Js$, e = 1.6 X $10^{-19} C$. $(Ans \quad V_a = 58.9 kV)$

3. An x-ray tube operates at $30 kV$ and the current through it is $2.0 mA$.

   Calculate the;

   (i) Electrical power input
   (ii) Number of electrons striking the target per second
   (iii) Speed of the electrons on striking the target if the electron mass is 9.11 X $10^{-31} kg$.

(iv) Minimum wavelength of the x-rays emitted $(P = 60Js^{-1} \quad n = 1.3\ x\ 10^{16}s^{-1} \quad u = 1.03\ x\ 10^{8}ms^{-1} \quad \lambda_{min} = 4.1\ x\ 10^{-11}m)$

4. The current in a water-cooled x-ray tube operating at 60kV is 30mA. 99% of the energy supplied to the tube is converted into heat at the target and is removed by flowing water at a rate of 0.060kgs^{-1}. Calculate the;

(i) rate at which energy is being supplied to the tube,

(ii) increase in temperature of the cooling water if the specific heat capacity of water is 4200$Jkg^{-1}K^{-1}$. $(P = 1800Js^{-1} \quad \Delta\theta = 7.1°C)$

5. The potential difference between the target and the cathode of an x-ray tube is 50kV and the current in the tube is 20mA. Only 1% of the total energy supplied is emitted as x-radiation.

(i) What is the maximum frequency of the x-rays produced,

(ii) At what rate must heat be removed from the target in order to keep it at a steady temperature? $(f_{max} = 1.21\ x\ 10^{19}Hz \quad P = 990Js^{-1})$

6. A 900W x-ray tube operates at a d.c potential difference of 30kV.

(i) Calculate the minimum wavelength of the x-rays

(ii) Calculate the current through the tube

(iii) If 99% of the power is dissipated as heat, estimate the number of x-ray photons produced per second.

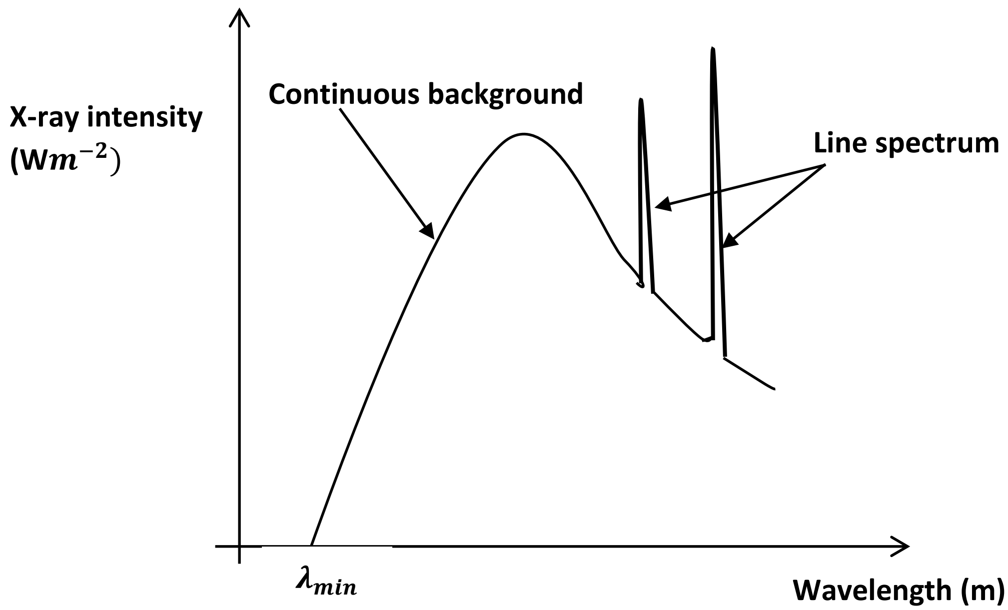
$(Ans \quad \lambda_{min} = 4.125\ x\ 10^{-11}m \quad I = 30mA \quad n = 1.875\ x\ 10^{15}s^{-1})$

X-RAY EMISSION SPECTRA.

A typical x-ray spectrum has two distinct components, that is;

- A background of continuous radiation, which depends on the operating tube voltage. It constitutes a continuous spectrum.
- A very intense emission at a few distinct wavelengths, called the x-ray line spectrum. The x-ray line spectrum depends on the nature of the target material.

A graph of intensity against wavelength of the x-ray.



The energy distribution from an x-ray tube, takes the form shown in the figure above, in which the line spectrum is super-imposed on a continuous spectrum.

A continuous spectrum has a definite cut-off wavelength, λ_{min} which increases to a maximum and then decreases gradually in the longer wavelengths. This is due to multiple collisions of the energetic electrons with the target atoms and being decelerated rapidly. It depends on the operating voltage of the tube.

A characteristic spectrum is a result of electron transitions within the atoms of the target material. At high enough tube voltages, the high energy bombarding electrons are able to penetrate deep into the atoms, and displace electrons from their inner shells. The displaced electrons can either be completely ejected out of the atom, or jump to the outer shells. The vacancy left in the inner shell makes the atom unstable and it said to be excited.

Stability is re-attained when an electron from the outer shell fills up the vacancy. However, this transition results into emission of photons of high frequency in the range of x-rays.

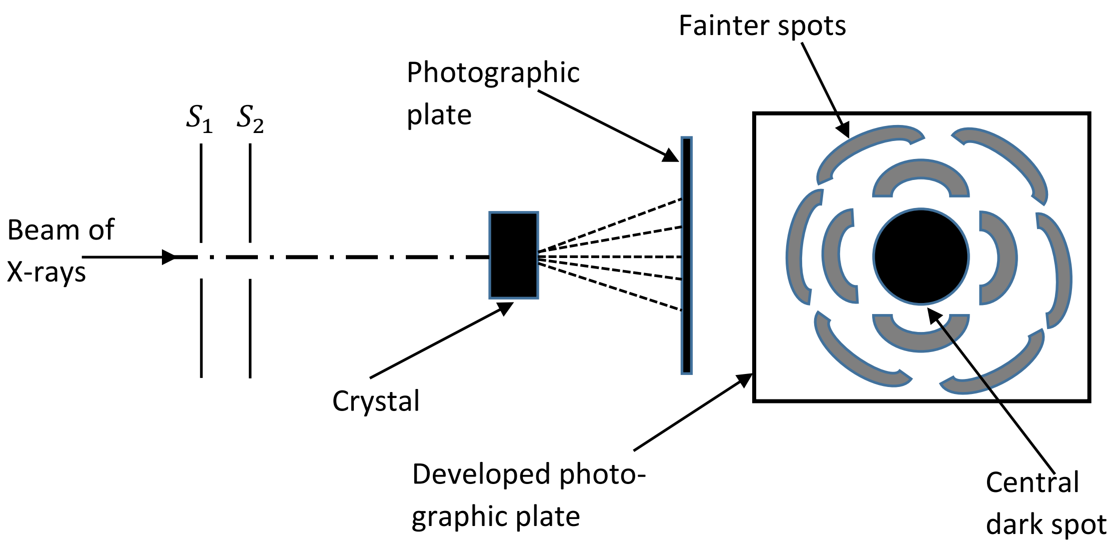
NOTE:

(i) **A line spectrum** refers to the discontinuous lines produced by electronic transitions with in the atoms, as electrons fall back to lower energy levels.

(ii) **Cut off wavelength** ($\boldsymbol{\lambda_{min}}$) is the minimum wavelength obtained when an energetic electron loses all its kinetic energy in a single encounter with the target atom. X-ray photon given off has maximum energy with the shortest possible wavelength, as; $\boldsymbol{eV_a = \frac{hc}{\lambda_{min}}}$

The Wave Nature of X-rays.

A narrow beam of monochromatic x-rays is incident on a crystal with dimensions compared to the wavelength of the x-rays. A photographic plate is then placed behind this arrangement, as shown below.



S_1 and S_2 are lead slits

The developed photographic plate reveals that most of the radiation passed straight through the crystal, to give a large central dark spot. However, some of the radiation produce a distinct pattern of fainter smaller spots around a central one. This shows that x-rays have been diffracted and confirms their wave nature.

NOTE:

X-rays are considered to be **electromagnetic waves** because;

(i) Their method of production involves accelerated charged particles.
(ii) They eject electrons from matter by photo-electric emission.
(iii) They give a line spectra.
(iv) They ionize gases through which they pass.

ATOMIC SPACING IN CRYSTALS.

Consider one unit of potassium chloride ($K^{+}Cl^{-}$).

Let M be the molecular mass in grams,

ρ is the density of $K^{+}Cl^{-}$

N_A is the Avogadro's number always equal to $6.02\ x10^{23}mol^{-1}$

The mass per molecule $= \frac{M}{N_A}$

Volume associated with one molecule $= \frac{M}{\rho N_A}$

Since one molecule of $K^{+}Cl^{-}$ has two atoms,

Volume associated with only one atom $= \frac{M}{2\rho N_A}$

However, if d is the inter-atomic spacing, it follows that

Volume associated with one molecule $= d\ x\ d\ x\ d$

$$= d^3$$

Hence, $\boldsymbol{d^3 = \frac{M}{2\rho N_A}}$

EXAMPLE:

Calculate the atomic spacing of sodium chloride if the relative atomic mass of sodium is 23.0 and that of chlorine is 35.5

$[the\ density\ of\ sodium\ hloride = 2.18\ x\ 10^3 kgm^{-3}]$.

$(Ans\quad d = 2.81\ x\ 10^{-10}m)$

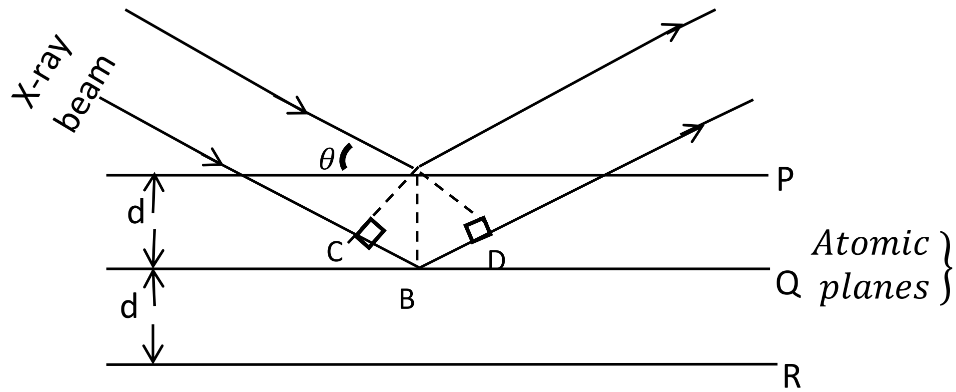
BRAGG'S LAW OF X-RAY DIFFRACTION.

Bragg discovered that a monochromatic beam of x-rays was reflected from a plane in a crystal which is rich in atoms, called an atomic plane.

A parallel beam of monochromatic x-rays is directed towards a crystal with inter- planar spacing of the same order as that of the wavelength of the x-rays.

When x-rays reflected from successive atomic planes are super-imposed, an interference pattern is formed.

The bright spots occur when the path difference between x-rays scattered by successive planes is an integral multiple of the wavelength.



For constructive interference to occur, the path difference is equal to the whole number multiple of the wavelength, that is;

$$CB + BD = n\lambda$$

$$d\sin\theta + d\sin\theta = n\lambda$$

$\boldsymbol{2d\sin\theta = n\lambda}$, where $n = 1, 2, 3, \ldots\ldots$ is the diffraction order.

Conditions for X-ray Diffraction to occur.

X-rays should have a wavelength of the same order as the inter-planar Spacing.

A parallel beam of x-rays must be incident on a consecutive atomic plane.

EXAMPLES:

1. An x-ray of wavelength $1\ x\ 10^{-10}m$ is required for the study of its diffraction in a crystal. Find the least accelerating voltage to be applied to an x-ray tube in order to produce these x-rays.

$(Ans \quad V_a = 1.24\ x\ 10^4 V)$

2. A beam of x-rays of wavelength $0.20nm$ is incident on a crystal at a glancing angle of $30°$. If the inter-planar separation is $0.20nm$, find the order of diffraction.

$(Ans \quad n = 1)$

3. A beam of x-rays of wavelength $8.24\ x\ 10^{-11}m$ is incident on a sodium chloride crystal of inter- planar separation $2.82\ x\ 10^{-10}m$. Calculate the first order diffraction angle.

$(Ans \quad \theta = 8.4°)$

4. A monochromatic x-ray beam of wavelength $1.0\ x\ 10^{-8}cm$ is incident on A set of planes in a crystal of spacing $2.8\ x\ 10^{-10}m$. What is the maximum Order possible with these x-rays?

$(Ans \quad n \approx 6)$

5. A beam of x-rays of frequency $3.56\ x\ 10^{18}Hz$ is incident on a potassium chloride crystal and the first order Bragg's reflection occurs at $7°41'$. If the density of potassium chloride is $1.98\ x\ 10^3 kgm^{-3}$ and its molecular mass is 74.6, calculate the value of Avogadro's number.

$(Ans \quad N_A = 6.385\ x\ 10^{23} mol^{-1})$

TRIAL QUESTIONS.

1. When fast moving electrons strike a metal target in an x-ray tube, two types of x-ray spectra are produced.
   (a) Draw a sketch graph of intensity against wavelength of the x-rays.
   (b) Account for the occurrence of the two types of spectra.

2. (a) State Bragg's law of X-ray diffraction.

(b) A beam of X-rays of wavelength $1.0\ x\ 10^{-10}m$ is incident on a set of cubic planes in a sodium chloride crystal. The first order diffracted beam is obtained for a grazing angle of $10.2°$. Find the;

(i) spacing between consecutive planes.

(ii) density of sodium chloride, if its molecular mass is 58.5

$(Ans \quad d = 2.824\ x\ 10^{-10}m, \quad \rho = 2.159kgm^{-3})$

3. (a) (i) Define the term X-rays

(ii) State four properties of X-rays

(b) A monochromatic beam of X-rays of wavelength $2.0\ x\ 10^{-10}m$ is incident on a set of cubic planes in a potassium chloride crystal. First order diffraction maxima are observed at a reflecting angle of $71.5°$. Find the density of potassium chloride given that its relative molecular mass is 74.5

$(Ans \quad \rho = 1.977\ x\ 10^{3}kgm^{-3})$

4. (a) (i) Draw a labelled diagram of an X-ray tube.

(ii) Use the diagram in (a) (i) above to describe how X-rays are produced.

(iii) State the energy changes that occur during the production of X-rays in an X-ray tube.

(b) When copper is used as a target of an X-ray tube, an intense monochromatic line is emitted. Its wavelength is measured by obtaining first order diffraction of $14°$ from a sodium chloride crystal, using Bragg's planes for which the spacing is $2.8\ x\ 10^{-10}m$.

Calculate the ;

(i) wavelength of the radiation

(ii) minimum p.d which can be used between the anode and the cathode of the X-ray tube.

$(Ans \quad \lambda = 1.355\ x\ 10^{-10}m \quad V_a = 9.133kV)$

5. (a) Explain briefly diffraction of X-rays by crystals and derive Bragg's law.

(b) (i) Give two conditions for X-ray diffraction to occur.

(ii) State two biological and two industrial uses of X-rays.

(c) In an X-ray tube, the electrons strike the target with a velocity of $3.75\ x\ 10^{-10}m$ after travelling a distance of $5.0cm$ from the Cathode. If a current of $10mA$ flows through the tube, find the;

(i) tube voltage

(ii) number of electrons striking the target per second.

(iii) number of electrons within a space of $1cm$ length between the anode and the cathode.

$(Ans\ V_a = 4003V,\ n = 6.25\ x\ 10^{16}s^{-1},\ n = 3.33\ x\ 10^{7}electrons)$

(d) A monochromatic beam of x-rays is incident on a certain crystal. At $0°$C, the first order diffraction maxima is observed at a glancing angle of $30.4°$. When the temperature of the crystal is raised to $400°$C, the first order maxima is observed at $30°$. Calculate the mean co-efficient of linear expansion of the crystal from a temperature range of $0°\text{C} - 400°\text{C}$.

$(Ans \quad \propto = 3.025\ x\ 10^{-5}K^{-1})$